A METHOD OF DETERMINING THE INTERPHASE HEAT-TRANSFER COEFFICIENT IN A FLUIDIZED BED OF GRANULAR MATERIALS

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As is well known, for a number of reasons there are considerable obstacles to determining the true heat-transfer coefficients in a fluidized bed. These include the difficulties in establishing the average motive force, i.e., reliable determination of the temperatures of the liquefying agent and the solid particles. In fact, an unsheathed thermocouple measures not the temperature of the liquefying agent of the solid particles but an intermediate value. At the same time, a sheathed thermocouple does not measure the exact temperature of the liquefying agent, due to heat losses and leakage through the sheathing system of the thermocouple.

A method is proposed below for determining the heat-transfer coefficient as an external problem assuming the solid phase is completely mixed and that the gaseous (liquid) phase moves with ideal displacement. With this method, which can also be used to determine the mass-transfer coefficient, it is not necessary to measure the temperatures of the liquefying agent and the solid particles in the bed. It is only necessary to determine the temperatures of the liquefying agent  $t_{in}$  at the inlet to the bed and of the solid particles  $\theta_0$  at the initial moment  $\tau = 0$ , and also the temperature of one of the phases ( $\theta$  or  $t_{out}$ ) at the outlet of the bed at an arbitrary time  $\tau$ .

Let us consider the unsteady heating (cooling) of a bed that operates without input and output of the solid phase. Let a liquefying agent (for example, a gas) in the amount of G kg/hr (with specific heat c) enter a fluidized bed containing  $G_T$  kg of solid particles (with specific heat  $c_T$ ). As a result of heat transfer with the gas at the initial temperature  $t_{in}$ , the bed is heated from temperature  $\theta_0$  at  $\tau = 0$ to  $\theta$  at time  $\tau$ .

Let us calculate the changes in the temperatures of the gas and solid material for a known contact surface  $F_p$  and average heat-transfer coefficient  $\alpha$  over  $F_p$ . At any time for an arbitrary unit particle surface dF included within a unit height of the bed, we can write

 $a\left(t-\theta\right)dF=-Gcdt,$ 

hence

$$\theta = t - \frac{Gc}{a} \frac{dt}{dF} .$$
 (2)

(1)

(3)

Let us differentiate this equation, assuming that the particle temperature is constant throughout the volume of the bed (and thus over F); that is,  $d\theta/dF = 0$ :

 $\frac{d^2t}{dF^2} + \frac{\alpha}{Gc} \frac{dt}{dF} = 0,$ 

hence

and

$$t = A \exp\left(-\frac{\alpha F}{Gc}\right) + B$$

$$\frac{dt}{dF} = -A \frac{a}{Gc} \exp\left(-\frac{aF}{Gc}\right).$$
(4)

The constants of integration A and B are determined from the following boundary conditions:

1) at F = 0,  $t = t_{in}$ ; then from (3)

$$t_{\rm in} = A + B; \tag{5}$$

2) from (1) and (4), we have

$$\frac{dt}{dF} = -\frac{a}{Gc} (t-\theta) = -A \frac{a}{Gc} \exp\left(-\frac{aF}{Gc}\right) ,$$

and then when F = 0

$$A = t_{\rm in} - \theta. \tag{6}$$

Combining Eqs. (5) and (6), we obtain  $B = \theta$ , and then from (3) we find t at time  $\tau$  after contact with the surface F:

$$t = (t_{in} - \theta) \exp(-\alpha F/Gc) + \theta =$$
  
=  $t_{in} \exp(-\alpha F/Gc) + \theta [1 - \exp(-\alpha F/Gc)].$  (7)

For the next calculation, we must know the instantaneous average over the entire surface  $F_D$  of the gas temperature  $t_{av}$ :

$$t_{av} = \frac{1}{F_p} \int_{F_p} tdF = \theta + (t_{in} - \theta) \left[1 - \exp\left(-\alpha F_p/Gc\right)\right] \frac{Gc}{\alpha F_p}.$$
 (8)

Hence, the average instantaneous motive force over  $F_p$  is

$$t_{av} - \theta = (t - \theta)_{av} = (t_{in} - \theta) \left[1 - \exp\left(-\alpha F_p/Gc\right)\right] \frac{Gc}{\alpha F_p}.$$
 (9)

Now let us consider the heating of the bed with time. For the unit interval  $\mathrm{d}\tau$ 

$$\alpha F_{\mathbf{p}}(t-\theta)_{\mathbf{av}}d\tau = G_{\mathbf{r}}c_{\mathbf{r}}d\theta,$$

Hence, considering (9), after integration we obtain

$$\ln \frac{t_{in} - \theta_0}{t_{in} - \theta} = \frac{Gc \tau}{G_r c_r} [1 - \exp(-\alpha F_p/Gc)].$$
(10)

This equation can be solved for  $\theta = \theta(\tau)$  and, with the aid of expression (7), also for  $t = t(\tau, F)$ :

$$\theta = t_{in} - (t_{in} - \theta_0) \exp\left\{-\frac{Gc\tau}{G_{\tau}c_{\tau}} \left[1 - \exp\left(-\alpha F_p/Gc\right)\right]\right\}, \quad (11)$$
$$t = t_{in} - (t_{in} - \theta_0) \left[1 - \exp\left(-\frac{\alpha F}{Gc}\right)\right] \times \\\times \exp\left\{-\frac{Gc\tau}{G_{\tau}c_{\tau}} \left[1 - \exp\left(-\frac{\alpha F_p}{Gc}\right)\right]\right\}. \quad (12)$$

By simple transformations we can also obtain an expression for the average motive force over time  $\tau$  corresponding to the over-all heat-transfer equation  $Q = \alpha F_D \Delta_{aV} \tau$ :

$$\Delta_{\mathbf{av}} = \frac{Gc}{\alpha F_{\mathbf{p}}} \left[ 1 - \exp\left(-\frac{\alpha F_{\mathbf{p}}}{Gc}\right) \right] \left(\theta - \theta_{\mathbf{0}}\right) / \left( \ln \frac{t_{\mathbf{in}} - \theta_{\mathbf{0}}}{t_{\mathbf{in}} - \theta} \right).$$
(13)

Expression (10) is most convenient for calculating  $\alpha$  from experimentally measured  $t_{in}$ ,  $\theta_0$ ,  $\tau$ , and  $\theta$ , and expression (12) is most convenient when calculating  $\alpha$  from  $t_{in}$ ,  $\theta_0$ ,  $\tau$ , and  $t = t_{out}$  when  $F = F_p$ . It is not difficult to solve these equations for  $\alpha$ . For example, from (10) we obtain

$$\alpha = \frac{Gc}{F_{\rm p}} \ln \left( 1 - \frac{G_{\rm r} c_{\rm r}}{Gc \tau} \ln \frac{t_{\rm in} - \theta_0}{t_{\rm in} - \theta} \right)^{-1}. \tag{14}$$

Thus, it is easy to calculate  $\alpha$  by measuring  $\theta$  or t at a known time  $\tau$  (in the former case, this can be done by cutting off the pressure at time  $\tau$ ; in the latter case, by measuring the gas temperature above the bed) and if we know  $\theta_0$  and  $t_{in}$ . The accuracy of this determina-

tion naturally depends upon the correctness of the model used (ideal gas displacement and complete mixing of the solid particles) and also upon the measurement method (the accuracy with which  $\tau$ , t<sub>in</sub>, and  $\theta_0$  are determined, elimination of the input effect, etc.). Moreover, the obtained  $\alpha$  values will come closer to the true values as the effects that result in effective  $\alpha$  are reduced (the longitudinal heat conductivity of the gas, discrepancy between the calculated and actual con-

tact surfaces, etc.). In particular, at high Reynolds numbers (Re > >  $10^2$ ), this method should give heat-transfer coefficients that are close to the true values.

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